Brain as a Quantum Neural Network

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Hudgkin-Huxley model [3] has been one of the most important system of differential equations in the field of neuroscience. The system of equation relates how action potential V_m evolves with time in neurons, given a current source $\mathcal{I}$.

$$
\begin{aligned}
C_m \frac{\partial V_m}{\partial t} &= \mathcal{I} - \bar{g}_K n^4 (V_m - V_K) - \bar{g}_{Na} n^4 (V_m - V_{Na}) - \bar{g}_l (V_m - V_l) \\
\frac{\partial n}{\partial t} &= \alpha_n(V_m)(1-n) - \beta_n(V_m) n \\
\frac{\partial m}{\partial t} &= \alpha_m(V_m)(1-m) - \beta_m(V_m) m \\
\frac{\partial h}{\partial t} &= \alpha_n(V_m)(1-h) - \beta_n(V_m) h
\end{aligned} \tag{1}
$$

where $\mathcal{I}$ is the ionic current, V_m is the membrane voltage, α, β are functions of V_m, C_m is the membrane capacitance, $\bar{g}_K, \bar{g}_{Na}, \bar{g}_l$ is the unit conductance of Potassium, Sodium and leaky channels respectively, V_K, V_{Na}, V_l is the voltage bias in the Potassium, Sodium and leaky channels respectively. The state variables (V_m, n, m, h) evolves where the input is the current $\mathcal{I}$. The above equation is a classical form of the Hudgkin-Huxley model. To incorporate in QNN framework, one needs to quantize the Hudgkin-Huxley model as performed by [2]. The authors in [2] considered a quantized Hodgkin-Huxley model based on a quantum-memristor formalism. It is to note that the quantized Hodgkin-Huxley model similar to its classical form will have four parameters namely, (V_m, n, m, h) which is a equal to the QNN framework when the number of pararmeters θ is four, corresponding to the Pauli matrices. In QNN, the parameters θ evolves as:

$$
\begin{aligned}
\frac{\partial \theta_\nu}{\partial t} = &-\sum_{\zeta=1}^{4}(\delta_{\zeta\nu} - \epsilon D^{\infty}_{\zeta\nu})\Big[\sum_{\mu,\nu,\delta,\gamma}^{4}\sum_{j,k,p,q}^{4} A^{\infty}_{j'kp'q}(\theta^*_\mu - \bar{\theta}^*_\mu)(\theta_\nu - \bar{\theta}_\nu)(\bar{\mathcal{G}}^{\delta}_{\zeta}(\theta_\gamma - \bar{\theta}_\gamma) \\
&+ \mathcal{G}^{\gamma}_{\zeta}(\theta^*_\delta - \bar{\theta}^*_\delta))\mathrm{Tr}\Big(B\sigma^\mu\alpha^j\alpha^k\sigma^\nu \otimes B\sigma^\delta\alpha^p\alpha^q\sigma^\gamma\Big) \\
&+ \eta\sum_{\delta,\gamma}^{4}\sum_{j,k}^{4} A^{\infty}_{j'k}(\bar{\mathcal{G}}^{\delta}_{\zeta}(\theta_\gamma - \bar{\theta}_\gamma) + \mathcal{G}^{\gamma}_{\zeta}(\theta^*_\delta - \bar{\theta}^*_\delta))\mathrm{Tr}\Big(B\sigma^\delta\alpha^j\alpha^k\sigma^\gamma\Big)\Big]
\end{aligned}
\tag{2}
$$

The parameter evolution given in QNN (2) can be corresponded to equation (1) where θ runs from $\theta_{1\to 4}$ as shown in [1]. Relating equation (1) and (2), will give us the quantity like matrix A^∞ and matrix $\mathcal{G}$ for the Hudgkin-Huxley model in brain, which will enable us to predict quantum chaos, quantum complexity in the model of brain. Physics since last century has created incredible tools to analyze and understand high energy components of universe like black holes, but it is also important and time to understand biophysical systems like brain using these revolutionary principles of physics. This maybe a possible roadmap to actually understand human brain particularly human consciousness which seems as an emergent phenomenon of choas [4], in every human being yet the most challenging question of the humankind.

References

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