Magnetohydrodynamics of nonlinear flow

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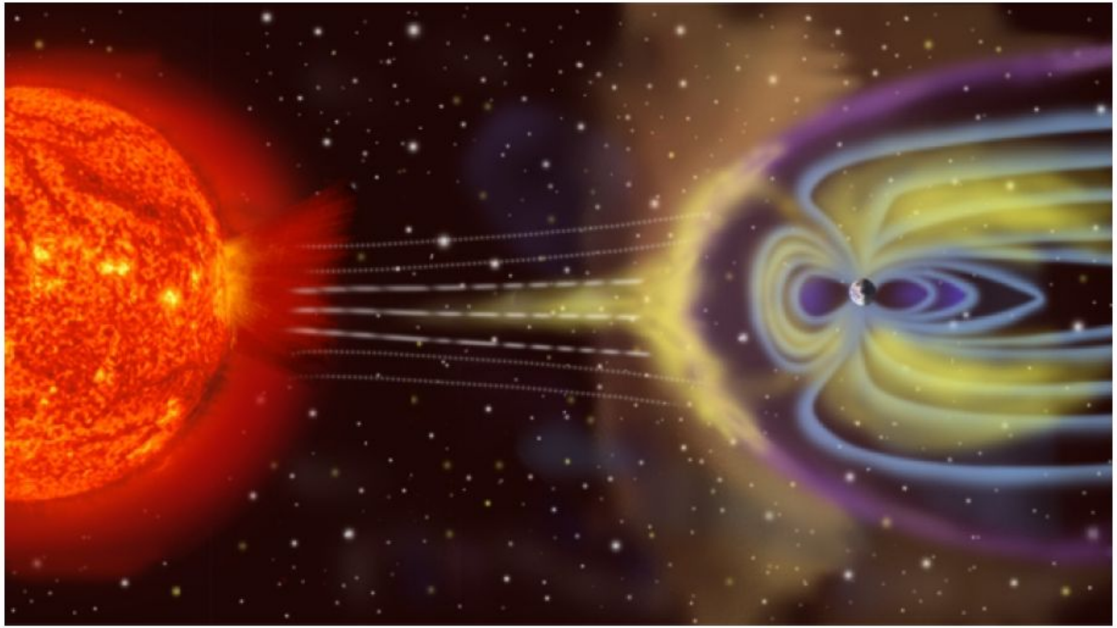
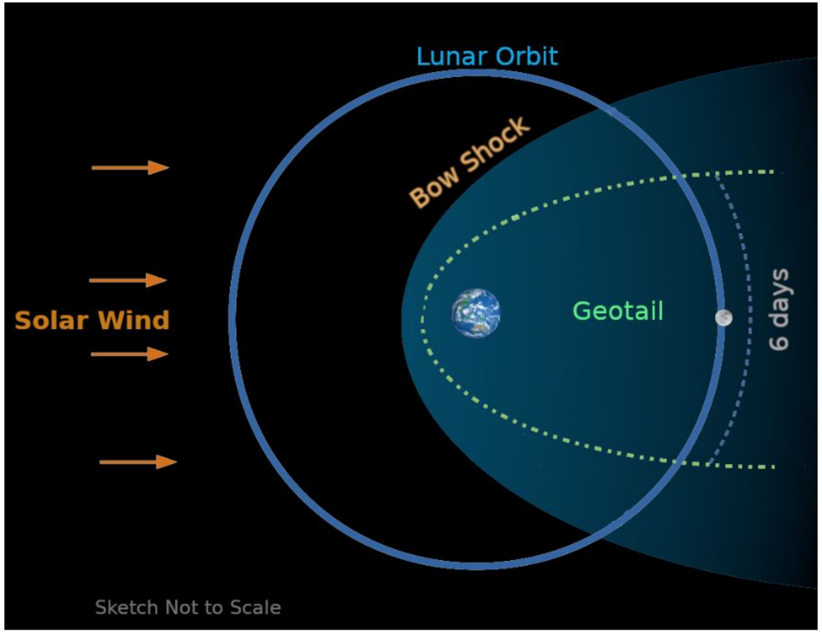
Reason for shielding of Solar wind



Earth’s magnetic field

Northern Lights

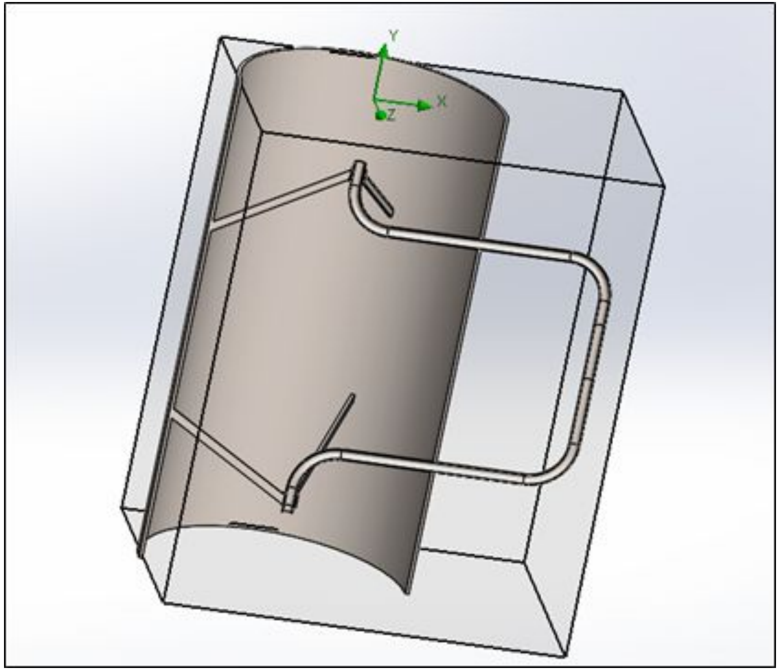


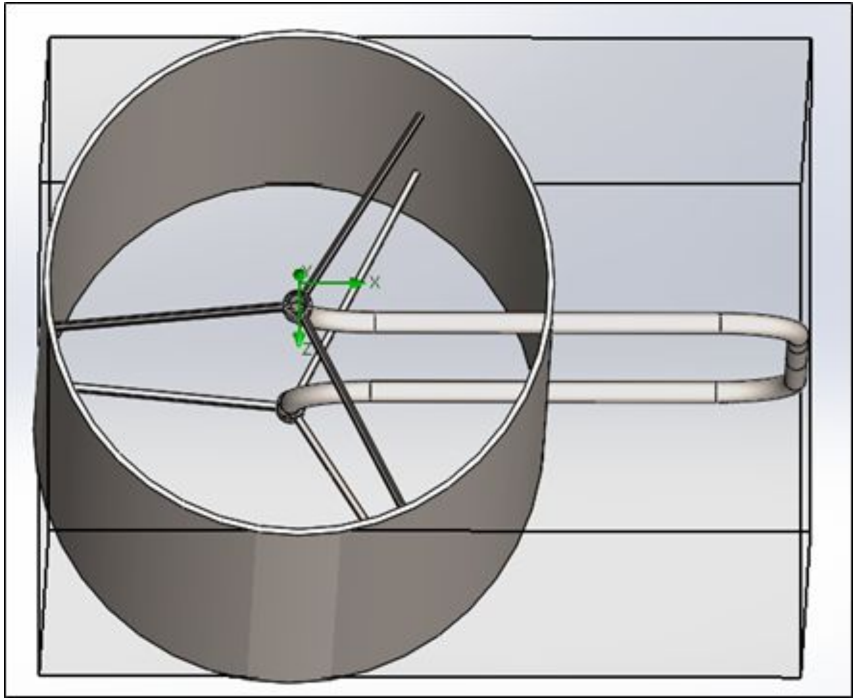
The CLASS instrument on Chandrayaan-2 is designed to detect direct signatures of elements present in the lunar soil. This is best observed when a solar flare on the Sun provides a rich source of x-rays to illuminate the lunar surface; secondary x-ray emission resulting from this can be detected by CLASS to directly detect the presence of key elements like Na, Ca, Al, Si, Ti and Fe.

Motivation

- ❖ Suction flooded vertical centrifugal pump having a bottom suction
- ❖ Suction line being 793NB direct measurement of discharge of liquid sodium vapor of 800K is impractical
- ❖ Shunt line of 26NB is coupled with main flow line , equipped with flow meter.
- ❖ Measured flow rate in shunt line - correlated with discharge in main flow line
- ❖ Flow fluctuations in shunt line, indicating amplified fluctuations in main line, increasing with discharge

SOLIDWORKS Modelling of Original Assembly

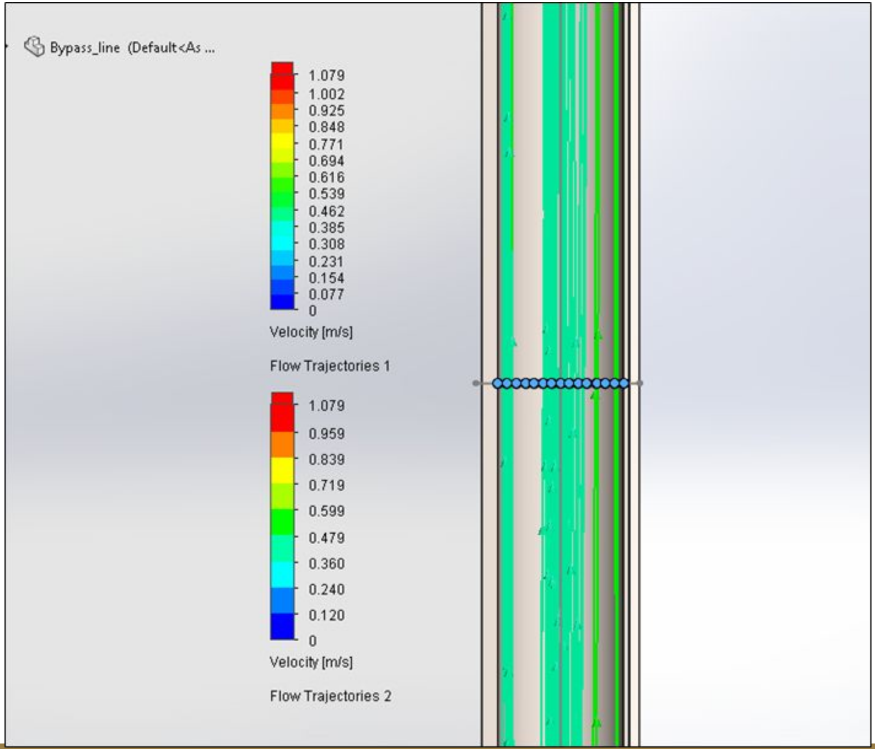




Boundary Conditions

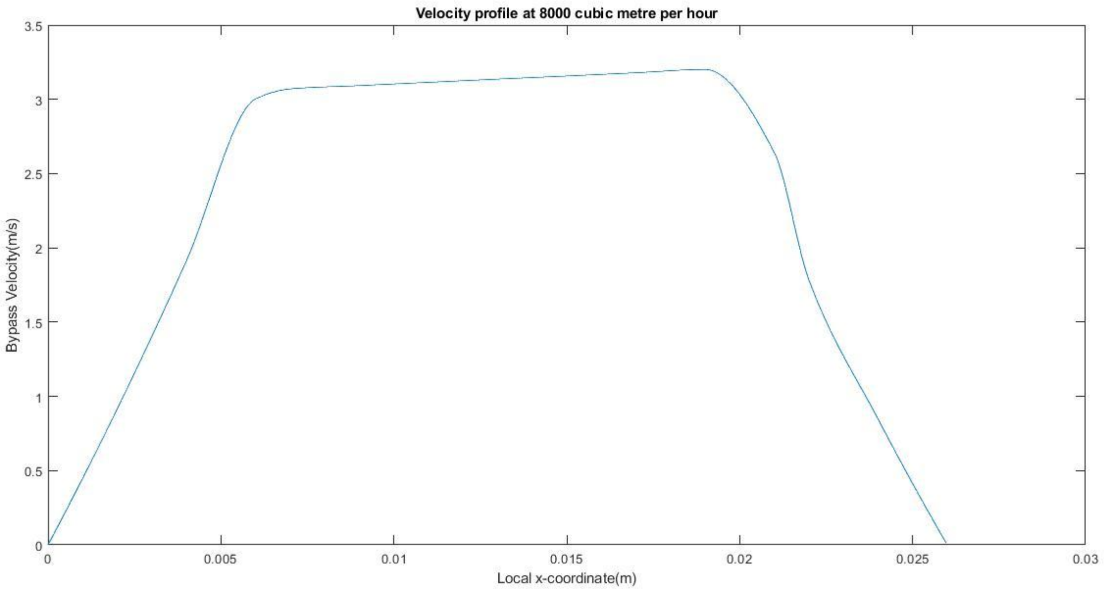
- Inlet: Inflow Volume Flow rate of Main Line
- Outlet: Open Boundary (Environmental Pressure)
- Real Walls: Shunt line Pipe, Gussets and Welds

Section at which Velocity Profile is taken (Flow Meter is placed)

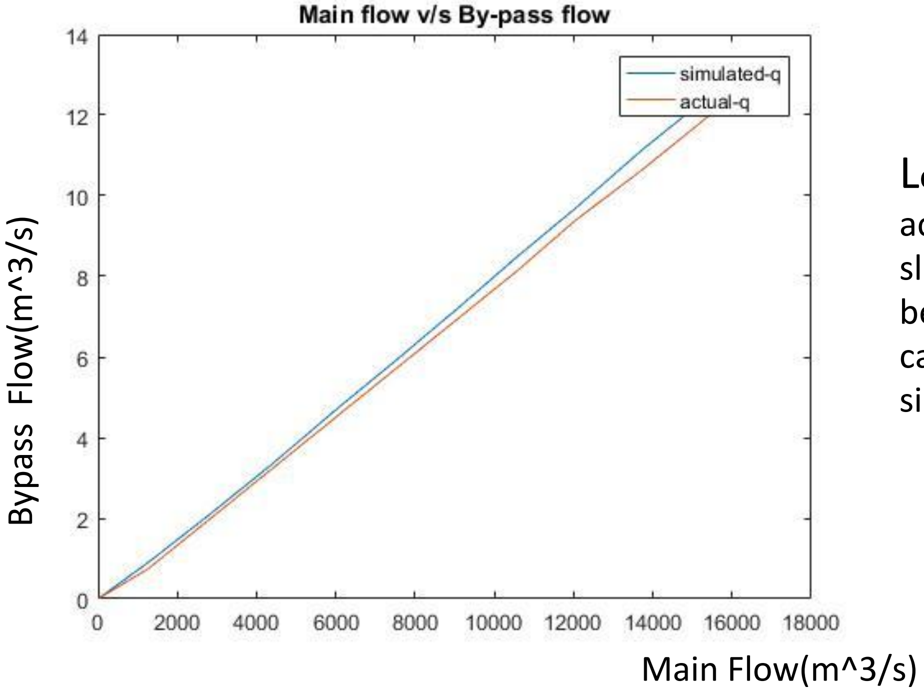


Simulated Velocity Profile of Bypass Line

- ➢ Navier-Stokes momentum & Continuity equation were solved



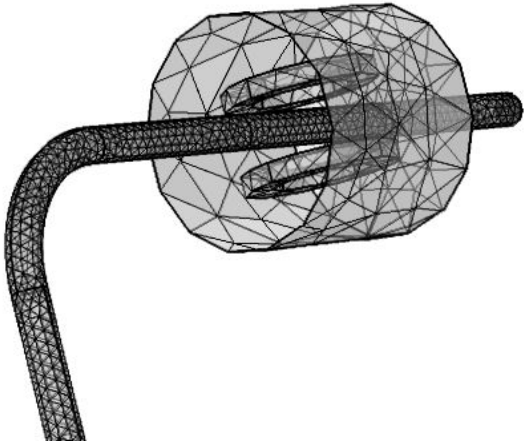
Comparison of Simulated and Calibrated Bulk Flow vs Bypass Flow

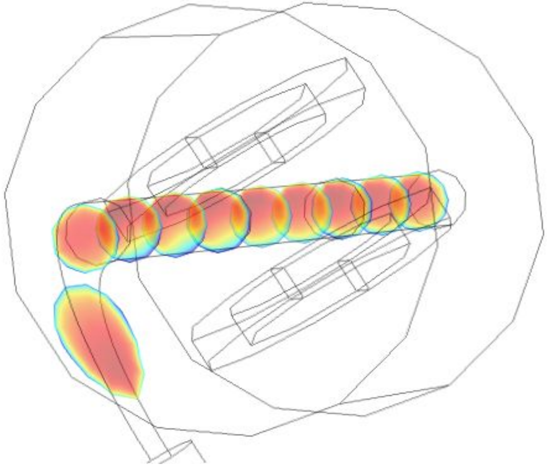


Lorentz Force is accountable for the slight deviation between the calibrated and the simulated curve

COMSOL MULTIPHYSICS Simulation of Bypass Line with Magnetic Field

- ➢ Coupled Algebraic yplus and RANS Turbulence model was used
- ➢ Physics-controlled meshing has been performed for easy convergence





Coupled Differential Equations for Magneto Hydrodynamics Flow

$$\Rightarrow \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} - \nu \Delta \mathbf{u} + \frac{1}{\rho} \nabla p = \frac{1}{\rho} \boldsymbol{J} \times \boldsymbol{B}$$

$$\Rightarrow \quad \nabla \cdot \mathbf{u} = 0$$

$$\Rightarrow \quad \boldsymbol{J} = \sigma \left(-\nabla V + \mathbf{u} \times \boldsymbol{B}_0 \right),$$

$$\Rightarrow \quad \nabla \cdot \boldsymbol{J} = 0,$$

Lorentz Force

$$\underbrace{\mathbf{J} \times \mathbf{B}}_{\text{Lorentz force}} = \underbrace{\boldsymbol{\kappa}\frac{B^2}{\mu_0}}_{\text{magnetic tension}} - \underbrace{\nabla_\perp\left(\frac{B^2}{2\mu_0}\right)}_{\text{magnetic pressure}}$$

Where ‘kappa’ is the curvature vector or rate at which tangent vector changes

Fluctuations in Potential measurement

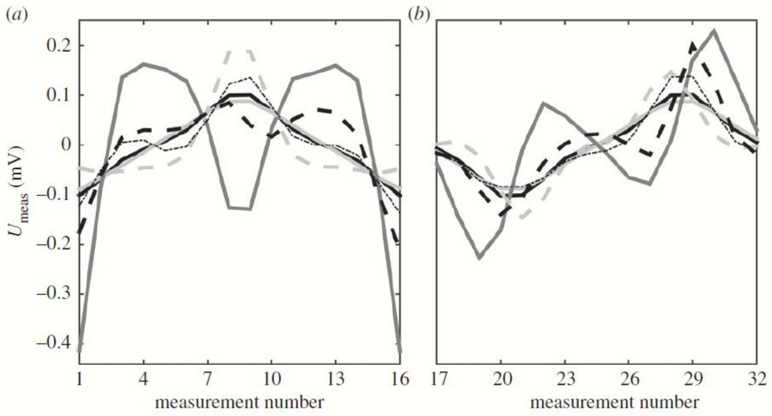


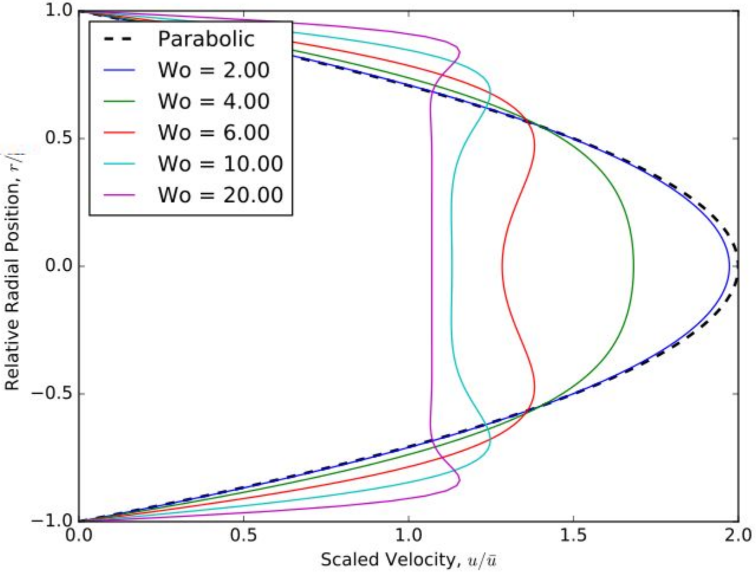
Figure 8. Simulated noisy voltage measurements between adjacent electrodes for Case 1 (solid black line), Case 2 (solid light grey), Case 3 (solid dark grey), Case 4 (dashed light grey), Case 5 (dashed black) and Case 6 (thin dash dotted black) corresponding to the first (*a*) and the second (*b*) magnetic field excitation.

Case 6	asymmetric CFD	flow after a pipe elbow

Lehtikangas *et al*, Reconstruction of velocity fields in electromagnetic flow tomography, Phil. Trans. R. Soc. A 374: 20150334.

Theoretical Velocity Profile of Pulsatile Flow (with Womersley Number ‘Wo’)

$$\mathrm{Wo} \equiv h\sqrt{\Omega/\nu},$$



Complex Analysis for Pulsatile Flow

$$u(r,t) = Re\left\{\sum_{n=0}^{N} \frac{i\,P'_n}{\rho\, n\, \omega}\left[1 - \frac{J_0(\alpha\, n^{1/2}\, i^{3/2}\, \frac{r}{R})}{J_0(\alpha\, n^{1/2}\, i^{3/2})}\right] e^{in\omega t}\right\}$$

where:

u	is the longitudinal flow velocity,
r	is the radial coordinate,
t	is time,
α	is the dimensionless Womersley number,
ω	is the angular frequency of the first harmonic of a Fourier
n	are the natural numbers,
P'_n	is the pressure gradient magnitude for the frequency nω,
ρ	is the fluid density,
μ	is the dynamic viscosity,
R	is the pipe radius,
$J_0(\cdot)$	is the Bessel function of first kind and order zero,
i	is the imaginary number, and
$Re\{\cdot\}$	is the real part of a complex number.

Fluctuations occur in Induced Electric Potential with the same frequency.

$$\hat{\mathbf{u}}(x_0,t) \equiv \sum_n \hat{\mathbf{u}}^{(n)}(x_0)\exp in\Omega t \quad \text{and} \quad \hat{p}(x_0,t) \equiv \sum_n \hat{p}^{(n)}(x_0)\exp in\Omega t$$

Maxwell's Equation

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t},$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \cdot \boldsymbol{D} = \rho_c.$$

Induction Equation: Magnetohydrodynamics

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = -\nabla \times \left(\frac{\boldsymbol{J}}{\sigma} - \mathbf{v} \times \boldsymbol{B} \right) = -\frac{1}{\sigma} \nabla \times \boldsymbol{J} + \nabla \times (\mathbf{v} \times \boldsymbol{B}).$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{V} \times \mathbf{B})}_{\text{convection}} + \underbrace{\frac{\eta}{\mu_0} \nabla^2 \mathbf{B}}_{\text{diffusion}}$$

Flow field remains along magnetic field line (Lundquist Theorem)

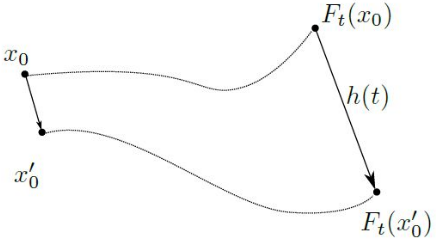
$$\partial_t \rho + \nabla \cdot (\rho u) = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{V} \times \mathbf{B})}_{\text{convection}}$$

Considering with resistivity is 0

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{B}}{\rho}\right) = \left(\frac{\mathbf{B}}{\rho} \cdot \boldsymbol{\nabla}\right) \mathbf{v},$$

Lundquist Theorem: Continued



$$\frac{d}{dt}h(t) = \mathrm{D}\big(t, x(t)\big)h(t).$$

$$\mathrm{D} = \frac{1}{2}\left[\nabla u + (\nabla u)^T\right].$$

h || B/ρ

Can initial small magnetic energy grow with time?

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{V} \times \mathbf{B})}_{\text{convection}} + \underbrace{\frac{\eta}{\mu_0} \nabla^2 \mathbf{B}}_{\text{diffusion}}$$

$$\frac{d}{dt}\frac{<B^2>}{2} =< \vec{B}\nabla\vec{v}\vec{B} > - \frac{\eta}{\mu_0} < |\vec{B}|^2 >$$ Averaged over volume

The field saturates to a value if it grows (How ?)

Magnetohydrodynamic Wave: Alfven wave

Small perturbations is introduced to magnetic field, density and pressure

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\boldsymbol{\nabla} \delta p + \frac{1}{4\pi} (\boldsymbol{\nabla} \times \delta \mathbf{B}) \times \mathbf{B}.$$

Alfven Wave: Continued

$$\rho\frac{\partial v_x}{\partial t} = -\frac{\partial \delta p}{\partial x} + \frac{B}{4\pi}(\frac{\partial \delta B_x}{\partial z} - \frac{\partial \delta B_z}{\partial x}), \qquad \rho\frac{\partial v_z}{\partial t} = -\frac{\partial \delta p}{\partial z},$$

$$\rho\frac{\partial v_y}{\partial t} = \frac{B}{4\pi}\frac{\partial \delta B_y}{\partial z},$$

$$\frac{\partial \delta B_x}{\partial t} = B\frac{\partial v_x}{\partial z}, \qquad \frac{\partial \delta B_z}{\partial t} = -B\frac{\partial v_x}{\partial x},$$

$$\frac{\partial \delta B_y}{\partial t} = B\frac{\partial v_y}{\partial z}.$$

Alfven Wave Equation

$$\left(\frac{\partial^2}{\partial t^2} - v_{\mathrm{A}}^2\frac{\partial^2}{\partial z^2}\right)(\delta B_y, v_y) = 0,$$

$$v_{\mathrm{A}} = \frac{B}{\sqrt{4\pi\rho}}$$

Literature References

1. Lehtikangas *et al*, Reconstruction of velocity fields in electromagnetic flow tomography, Phil. Trans. R. Soc. A 374: 20150334.
2. Bacciotti *et al,* Axisymmetric magnetohydrodynamic equations: Exact solutions for stationary incompressible flows, Physics of Fluids B: Plasma Physics (1989-1993) 4, 35 (1992);
3. Hazarika *et al*, Effect of Applied Magnetic Field on Pulsatile Flow of Blood in a Porous Channel, IJSIMR Volume 2, Issue 7, July 2014, PP 675-683
4. He *et al*, Simple Calculation of the Velocity Profiles for Pulsatile Flow in a Blood Vessel Using Mathematica, Annals of Biomedical Engineering, Vol. 21, pp. 45-49, 1993
5. Walker *et al*, Pulse Propagation in Fluid-Filled Tubes, Journal of Applied Mechanics

Thank you